

Departement of Computer Science

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Algorithms & Data Structures**Exercise sheet 1****HS 26**

The solutions for this sheet are submitted on Moodle until 27 September 2026, 23:59.

Exercises that are marked with a * are more challenging. We recommend first focusing on the other exercises.

You can use results from previous parts without solving those parts.

Exercise 1.1 *Sum of Squares (2 bonus points).*

Prove by mathematical induction that for every positive integer n ,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Recall the notation used here: $\sum_{i=1}^n i^2 = 1^2 + 2^2 + \dots + n^2$.

Exercise 1.2 *Sum of reciprocals of roots.*

Consider the following claim:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \leq \sqrt{n}.$$

A student provides the following induction proof. Is it correct? If not, explain where the mistake is.

Base case: $n = 1$,

$$\frac{1}{\sqrt{1}} \leq 1, \text{ which is true.}$$

Induction hypothesis: Assume the claim holds for $n = k$, i.e.

$$\frac{1}{\sqrt{1}} + \dots + \frac{1}{\sqrt{k}} \leq \sqrt{k}.$$

Induction step: Then, starting from the claim we need to prove for $n = k + 1$ and using logical

equivalences:

$$\begin{aligned} \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \leq \sqrt{k+1} &\iff \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{k}} \leq \sqrt{k+1} - \frac{1}{\sqrt{k+1}} \\ &\iff \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{k}} \leq \frac{k+1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+1}} \\ &\iff \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{k}} \leq \frac{k}{\sqrt{k+1}} \leq \frac{k}{\sqrt{k}} \leq \sqrt{k}, \end{aligned}$$

which is true, therefore the claim holds by the principle of mathematical induction.

Exercise 1.3 Asymptotic growth – part i (2 bonus points).

Recall the concept of asymptotic growth that we introduced in Exercise sheet 0: If $f, g : \mathbb{N}_{\geq 10} \rightarrow \mathbb{R}^+$ are two functions, then:

- We say that f grows asymptotically slower than g if $\lim_{m \rightarrow \infty} \frac{f(m)}{g(m)} = 0$. If this is the case, we also say that g grows asymptotically faster than f .

Prove or disprove each of the following statements.

- $f(m) = 100m^3 + 10m^2 + m$ grows asymptotically slower than $g(m) = 0.001 \cdot m^5$.
- $f(m) = \log(m^3)$ grows asymptotically slower than $g(m) = (\log m)^3$.
- $f(m) = e^{2m}$ grows asymptotically slower than $g(m) = 2^{3m}$.

Hint: Recall that for all $n, m \in \mathbb{N}$, we have $n^m = e^{m \ln n}$.

Exercise 1.4 Asymptotic growth – part ii.

Prove or disprove each of the following statements.

- * If $f(m), g(m) > 1$ and $f(m)$ grows asymptotically slower than $g(m)$, then $\log(f(m))$ grows asymptotically slower than $\log(g(m))$.
- * $f(m) = \ln(\sqrt{\ln(m)})$ grows asymptotically slower than $g(m) = \sqrt{\ln(\sqrt{m})}$.

Hint: You can use L'Hôpital's rule from sheet 0.

Exercise 1.5 Proving Inequalities.

- Prove the following inequality by mathematical induction

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{2n-1}{2n} \leq \frac{1}{\sqrt{3n+1}}, \quad n \geq 1.$$

In your solution, you should address the base case, the induction hypothesis and the induction step.

- * Replace $3n+1$ by $3n$ on the right side, and try to prove the new inequality by induction. This inequality is even weaker, hence it must be true. However, the same induction-step estimate fails. Try to explain to yourself how is this possible?